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Abstract:
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Index Terms:
computational complexity conductors (electric) conjugate gradient methods electric fi electrical engineering electrical engineering computing electromagnetic wave scatterin integral equations magnetic fields matrix multiplication 3D conducting bodies CFIE MFIE algorithm complexity combined field integral equation computational complexity conjugate gradient method electric field integral equation electromagnetic scattering f multipole method solution iterative method magnetic field integral equation matrix eq matrix-vector multiplication multilevel fast multipole algorithm nonnested method ray-propagation fast multipole algorithm

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FAST MULTIPOLE METHOD SOLUTION OF THREE DIMENSIONAL INTEGRAL EQUATION †

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1. Introduction

The fast multipole method (FMM) [1-6] speeds up the matrix-vector multiply in the conjugate gradient (CG) method when it is used to solve the matrix equation iteratively. In this paper, FMM is applied to solve the electromagnetic scattering from three dimensional arbitrary shape conducting bodies. The electric field integral equation (EFIE), magnetic field integral equation (MFIE), and combined field integral equation (CFIE) are considered. FMM formula for CFIE has been derived, which reduces the complexity of a matrix-vector multiply from $O(N^2)$ to $O(N^{1.5})$, where N is the number of unknowns. With a nonnested method, using the ray-propagation fast multipole algorithm (RPFMA), the cost of a FMM matrix-vector multiply is reduced to $O(N^{4/3})$. We have implemented a multilevel fast multipole algorithm (MLFMA), whose complexity is further reduced to $O(N\log N)$. The FMM also requires less memory, and hence, can solve a larger problem on a small computer.

2. The Fast Multipole Method (FMM)

Practical electromagnetic problems are often three-dimensional and involve arbitrary geometry. The arbitrary surface is described by dividing it into a number of connected patches which are mathematically described as parametric quadratic surfaces [7]. For conducting objects, the electric field integral equation (EFIE) is given by

$$\hat{t}\cdot\int_S \overline{\mathbf{G}}(\mathbf{r},\mathbf{r}')\cdot\mathbf{J}(\mathbf{r}')dS' = \frac{4\pi i}{k\eta}\hat{t}\cdot\mathbf{E}^i(\mathbf{r}), \tag{1}$$

and magnetic field integral equation (MFIE) for closed conducting objects is given by

$$2\pi\hat{t}\cdot\mathbf{J}(\mathbf{r}) - \hat{t}\cdot\hat{n}\times\nabla\times\int_s dS' g(\mathbf{r},\mathbf{r}')\mathbf{J}(\mathbf{r}') = 4\pi\hat{t}\cdot\hat{n}\times\mathbf{H}^i(\mathbf{r}), \tag{2}$$

where

$$\overline{\mathbf{G}}(\mathbf{r},\mathbf{r}') = (\overline{\mathbf{I}} - \frac{1}{k^2}\nabla\nabla')g(\mathbf{r},\mathbf{r}'), \qquad g(\mathbf{r},\mathbf{r}') = \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}. \tag{3}$$

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Then combined field integral equation (CFIE) for closed conducting objects is simply a linear combination of EFIE and MFIE. The integral equations are approximated by matrix equations using the method of moments (MOM) with specially designed basis functions for subdomains which contain surface curvature. The basis functions are the generalized rooftop functions.

The FMM idea is first to divide the subscatterers into groups. Then, addition theorem is used to translate the scattered field of different scattering ~~centers within a group into a single center.~~ Hence, the number of scattering centers is reduced. Similarly, for each group, the field scattered by all the other group centers can be first "received" by the group center, and then "redistributed" to the subscatterers belonging to the group.

~~After some derivations [2,6]~~ using the addition theorem, we can rewrite the matrix-vector multiplication as

$$\sum_{i=1}^{N} A_{ji}a_i = \sum_{m'\in B_m}\sum_{i\in G_{m'}} A_{ji}a_i + \frac{ik}{4\pi}\int d^2\hat{k}\,\mathbf{V}_{fmj}(\hat{k})\cdot \sum_{m'\notin B_m} \alpha_{mm'}(\hat{k}\cdot\hat{r}_{mm})\sum_{i\in G_{m'}} \mathbf{V}^{*}_{sm'i}(\hat{k})a_i, \tag{4}$$

where

$$\alpha_{mm'}(\hat{r}_{mm'}\cdot\hat{k}) = \sum_{l=0}^{L} i^l(2l+1)h_l^{(1)}(kr_{mm'})P_l(\hat{r}_{mm'}\cdot\hat{k}) \tag{5}$$

$$\begin{aligned}\mathbf{V}_{fmj}(\hat{k}) &= \alpha\int_S dSe^{i\mathbf{k}\cdot\mathbf{r}_{jm}}(\bar{\mathbf{I}}-\hat{k}\hat{k})\cdot\mathbf{t}_j(\mathbf{r}_{jm})\\ &\quad -(1-\alpha)\hat{k}\times\int_S dSe^{i\mathbf{k}\cdot\mathbf{r}_{jm}}\mathbf{t}_j(\mathbf{r}_{jm})\times\hat{n}\end{aligned} \tag{6}$$

$$\mathbf{V}_{sm'i}(\hat{k}) = \int_S dSe^{i\mathbf{k}\cdot\mathbf{r}_{im'}}(\bar{\mathbf{I}}-\hat{k}\hat{k})\cdot\mathbf{j}_i(\mathbf{r}_{im'}) \tag{7}$$

The first term in (4) is the contribution from nearby groups (including the self-group), and the second-term is the far interaction calculated by FMM. The computation cost using (4) with 2-level FMM is in order $O(N^{1.5})$.

To implement a multilevel fast multipole algorithm (MLFMA), the entire object is first enclosed into a large cube, which is partitioned into eight smaller cubes. Each subcube is then recursively subdivided into smaller cubes until the edge length of the finest cube is about half of a wavelength. When the cube becomes larger from the finest level to the coarsest level, the numbers of multipole expansions should increase. In the first sweep, the outer multipole expansions are computed at the finest level, then the expansions for larger cube are obtained using shifting and interpolation. At the coarsest level, the local multipole expansions contributed from well-separated cubes are calculated using the second part of (4). At the second sweep, the local expansions

for smaller cube include the contributions from parent cube using shifting and anterpolation, and from the well-separated cube at this level but not well-separated at the parent level. At the finest level, the contributions from non-well-separated cube are calculated directly. Since only nonempty cubes are considered, the complexity for MLFMA is further reduced to $O(N \log N)$.

3. Results and Conclusions

Figure 1 shows the validation of the numerical result from combined field integral equation (CFIE) with FMM against the Mie series solution of the bistatic RCS of a metallic sphere of radius 1m at frequency of 0.72GHz for the parallel polarization. 9408 unknowns with 2-level FMM are used. The solutions of CFIE with FMM agree with Mie series very well.

Figure 2 shows the bistatic RCS of a one meter long metallic square plate at 4.5GHz in the xy plane with incident angle $\theta = 45°$. 32512 unknowns with 6-level FMM are used. The calculation is done by solving EFIE on a SUN-SPARC 2 with 64MB RAM. There is a good agreement between our results and the approximation by physical optics when the RCS is bigger than 0 dB.

In conclusion, the fast multipole method (FMM) has been implemented to speed up the matrix-vector multiply in the CG method when it is used to solve EFIE, MFIE, and CFIE. At all frequencies, CFIE has an unique solution, and converges faster than EFIE and MFIE since the matrix from CFIE has a smaller condition number than those from EFIE and MFIE. FMM approach reduces the complexity of a matrix-vector multiply from $O(N^2)$ to $O(N^{1.5})$. With a multilevel fast multipole algorithm (MLFMA), the complexity is further reduced to $O(N \log N)$. The FMM also requires less memory, and hence, can solve a larger problem on a small computer.

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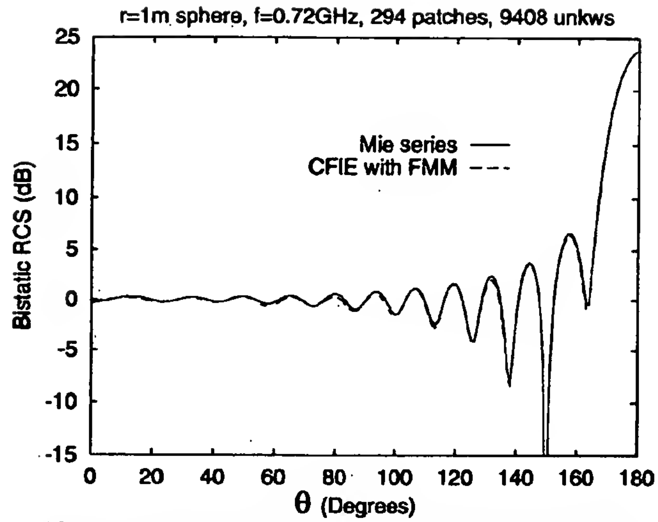

Figure 1. Validations of CFIE with FMM against the Mie series of the bistatic RCS of a metallic sphere of radius 1m at 0.72GHz for VV polarization. The RCS is normalized by πa^2.

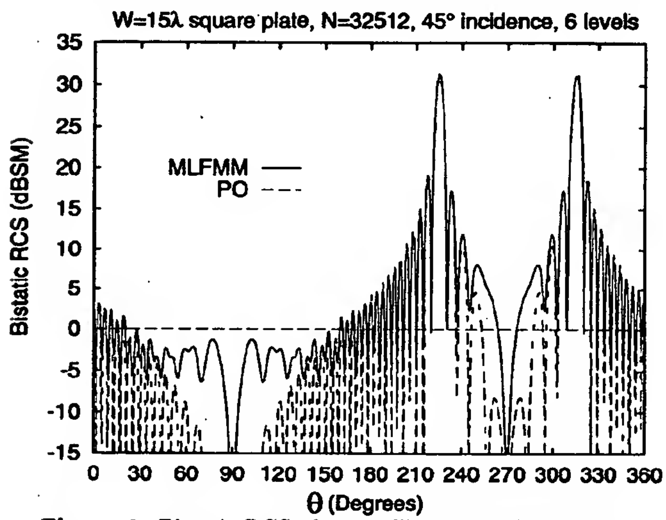

Figure 2. Bistatic RCS of a metallic square plate of length 1m at 4.5GHz for VV polarization with 45° incident angle.